

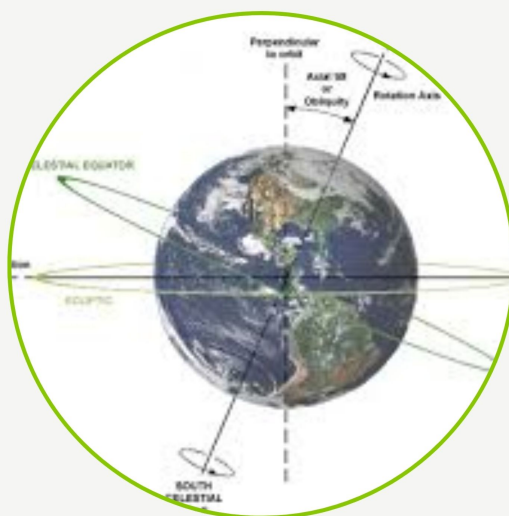
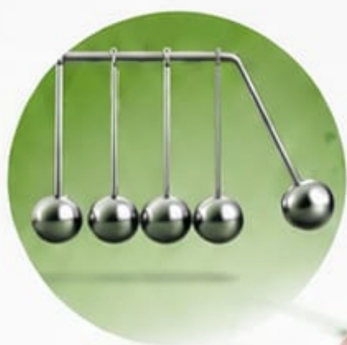


EXAM JEETOO
LEARN.TEST.ACHIEVE
CONCEPTCRACKS SMAARTCLASS PVT.LTD.

MOTION

FOUNDATION COURSE

CLASS IX-X



1

Motion



"Motion is change of position of an object with time."

1. Motion

If we look around us, we find that there are number of objects which are in motion.

An object is said to be in motion if it changes its position with the passage of time.

Now observe the following bodies or objects to understand the meaning of the term "motion". Cars, cycles, motorcycles, scooters, buses, rickshaws, trucks, etc. running on the road, birds flying in the sky, fish swimming in water, all these objects are in motion. Very small objects like atoms, molecules and very large objects like planets, stars and galaxies are also in motion.

Thus, all objects ranging from the smallest atom to the largest galaxy are in continuous motion.

Kinematics is the science of describing the motion of objects using words, diagrams, numbers, graphs and equations.

"Motion is the change in position of an object with time."

Concept of a point object (or particle)

Point object

An extended object can be treated as a point object when the distance travelled by the object is much greater than its own size.

A point object (or particle) is one which has no linear dimensions but possesses mass.

Examples : (i) Study of motion of a train travelling from Kota to New Delhi. (ii) Revolution of earth around the sun for one complete revolution.

Describing motion

When a tree is observed by an observer A standing at the railway station, the tree is at rest. This is because position of the tree is not changing with respect to the observer A (see figure).



Describing motion

Now, when the same tree is observed by an observer B sitting in a superfast train moving with a velocity v , then the tree is moving with respect to the observer because the position of tree is changing with respect to the observer B.

Rest and motion are relative terms : There is nothing like absolute rest or motion. This means that an object can be at rest and also in motion at the same time i.e. all objects, which are stationary on earth, are said to be at rest with respect to each other, but with respect to the sun they are making revolutions. In order to study motion, therefore, we have to choose a fixed position or point with respect to which the motion has to be studied. Such a point or fixed position is called a **reference point** or the **origin**.



Discuss whether the walls of your classroom are at rest or in motion.

Explanation

The walls of our classroom are at rest with respect to the ground or earth. But, they are in motion with respect to an object or an observer outside the earth. This is because the earth is moving about its own axis as well as it is revolving around the sun. Thus, the state of rest and motion are not absolute, they are relative terms.

2. Scalar and vector quantities

Scalar quantity

A physical quantity that is defined by its magnitude only is called a scalar quantity.

Examples : Mass, time, distance, speed, work, power, energy, electric charge, volume, density, pressure, electric potential, temperature, etc.

Scalar quantities follow the algebraic (scalar) laws of addition.

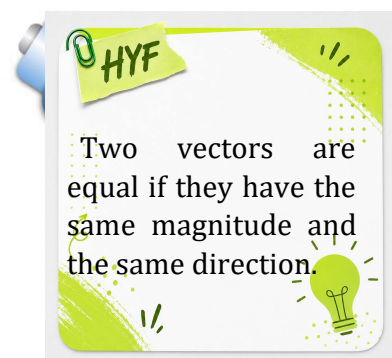
Vector quantity

A physical quantity that is defined by its magnitude as well as direction is called a vector quantity.

Examples : Velocity, acceleration, force, displacement, momentum, weight, torque, electric field, magnetic field, etc.

Vector quantities follow the vector laws of addition.

Arrows (or rays) are used to represent vectors. The direction of the arrow gives the direction of the vector. The length of the arrow is proportional to the magnitude of the vector.



Difference between scalar & vector quantities

	Scalar quantities	Vector quantities
1	These quantities are completely specified by their magnitude only.	These quantities are completely specified by their magnitude as well as direction.
2	These quantities change by change in their magnitude only.	These quantities change by change in either their magnitude or direction or both.
3	These quantities are added or subtracted by laws of ordinary algebra.	These quantities are added or subtracted by laws of vector addition.

3. Distance and displacement

Distance

The length of the actual path between the initial and the final position of a moving object in the given time interval is known as the distance travelled by the object.

Distance = Length of path I (ACB) (see figure)

Distance is a **scalar** quantity. It is always taken positive.

Distance is measured by odometer in vehicles.

Units

In SI system : metre (m).

In CGS system : centimetre (cm).

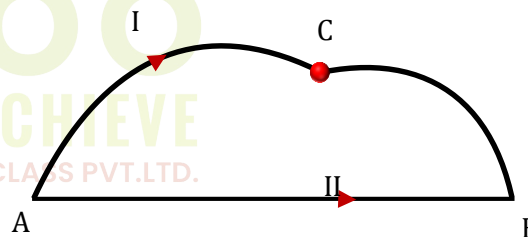
Displacement

The shortest distance between the initial position and the final position of a moving object in the given interval of time is known as the displacement of the object.

Displacement = Length of path II (AB) (see figure)

Displacement of an object may also be defined as the change in position of the object in a particular direction. That is,

Displacement of an object = Final position – Initial position of the object = $x_f - x_i$



Concept of distance and displacement

- ☞ Displacement is a **vector** quantity.
Displacement can be positive, negative or zero.

Units

In SI system : metre (m)

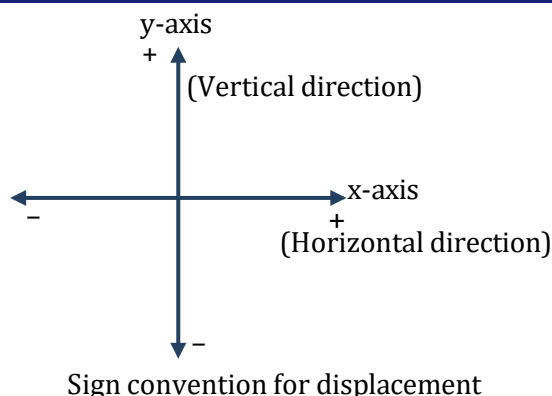
In CGS system : centimetre (cm)

Important points related to distance and displacement

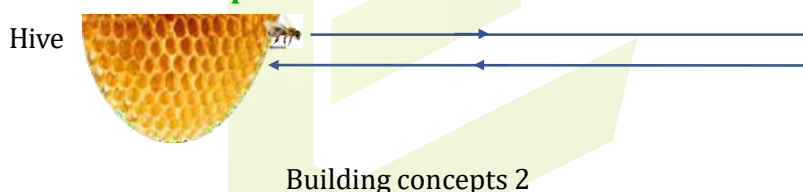
During motion, displacement of an object may be zero but the distance travelled by the object is never zero.

Distance travelled by an object is either equal to or greater than the magnitude of displacement of the object.

- ☞ Distance is equal to magnitude of displacement when a body moves in a straight line in a particular direction or it is in uniform motion.



A honeybee leaves the hive and travels 2 m as it returns to the hive. Is the displacement for the trip the same as the distance travelled? If not, why not?



Explanation

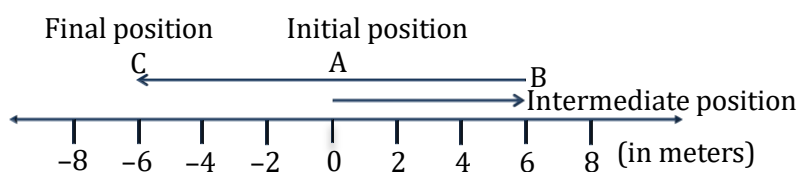
No, the displacement and the distance are not same. This is because the displacement is the change of position of object in motion while distance is length of path travelled by it.

Here, the distance travelled = 2 m

While the displacement = 0, because the position of honey bee is not changed.



- 1. Motion of a particle is shown below on a number line. Find the displacement from a) A to B (b) B to C (c) overall journey. Also, find distance for overall journey.**



Numerical Ability 1 (1)

Solution

- (a) At point A, initial position of the particle (x_i) is 0 m and at point B, the final position of the particle (x_f) is +6 m.

$$\text{Displacement from A to B} = x_f - x_i = (6) - (0) = \mathbf{+6 \text{ m}}$$

- (b) At point B, initial position of the particle (x_i) is 6 m and at point C, the final position of the particle (x_f) is -6 m.

$$\text{Displacement from B to C} = x_f - x_i = (-6) - (6) = \mathbf{-12 \text{ m}}$$

- (c) At point A, initial position of the particle (x_i) is 0 m and at point C, the final position of the particle (x_f) is -6 m.

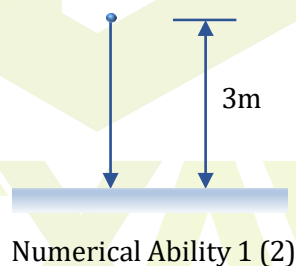
$$\begin{aligned} \text{Displacement of overall journey (i.e. A to B, B to C)} \\ = x_f - x_i = (-6) - (0) = \mathbf{-6 \text{ m}} \end{aligned}$$

Here, distance can also be found by adding positive values of displacement AB & displacement BC. i.e.,

$$\begin{aligned} \text{Distance travelled during overall journey} &= AB + BC \\ &= 6 + 12 = \mathbf{18 \text{ m}} \end{aligned}$$

Here Distance > |Displacement|

2. A body falls from a height of 3 m. Find displacement and distance.

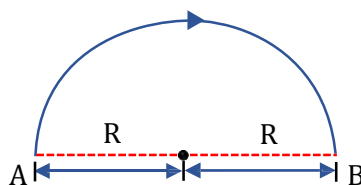
**Solution**

The initial position of the particle (x_i) is 3 m (the body is 3 m above the ground) and the final position of the particle (x_f) is 0 m. (see figure)

$$\text{Displacement} = x_f - x_i = 0 - 3 = \mathbf{-3 \text{ m}},$$

$$\text{Distance} = 3 \text{ m (see figure)}$$

3. A particle moves along a circular path as shown in figure. Find distance travelled and displacement.

**Decode the problem**

To find the displacement, we should know the initial and the final position of the particle.

Apply the formula

$$\text{Displacement} = \mathbf{x_f - x_i}$$

Distance = Length of actual path travelled

Solution

The shortest distance is the straight line between points A and B.

Displacement = diameter AB = $2R$

To find the distance, we should know the formula of the circumference of the circle.

Circumference of the circle = $2\pi r$

$$\begin{aligned}\text{Distance travelled} &= \frac{1}{2} \times (\text{circumference of the circle}) \\ &= \frac{1}{2} (2\pi R) = \pi R\end{aligned}$$



- ★ Always take care of the sign convention while finding the displacement of the body.



- ★ Always take care of the direction (north, south, east, west) given in the problem.

- ★ **For a Circle :** Diameter = $2r$

$$\text{Circumference} = 2\pi r \text{ or } \pi d \left(\pi = \frac{22}{7} \text{ or } 3.14 \right)$$

- ★ **Quadrilateral**

Perimeter of quadrilateral = AB + BC + CD + DA

Perimeter of rectangle = $2(l + b)$

Perimeter of square = $4 \times \text{side}$

- ★ **Pythagoras theorem**

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$AC^2 = BC^2 + AB^2$$

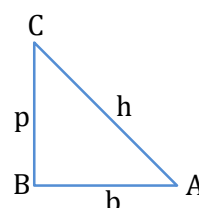
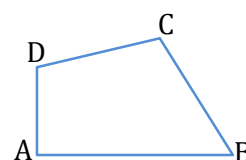
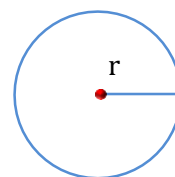
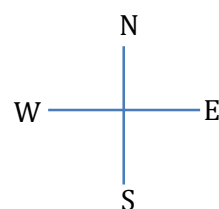
Decode the problem

To find the displacement, we should know the initial and the final position of the particle.

Apply the formula

Displacement = $x_f - x_i$

Distance = Length of actual path travelled




**Quick
Tips**

- ★ Whenever a particle changes its direction, distance is greater than displacement.
- ★ A body moving in a circular path when reaches its original position after one round, then the displacement at the end of one round is zero, but the distance travelled by it is equal to the circumference of circular path.
- ★ A body moving in a circular path when covers $\frac{1}{4}$ th of the circle then, the distance travelled by it is $\frac{vr}{2}$ and displacement is $\sqrt{2}r$.
- ★ A body moving in a circular path when covers $\frac{3}{4}$ th of the circle then, the distance travelled by it is $\frac{3vr}{2}$ and displacement is $\sqrt{2}r$.

Comparison between distance and displacement

	Distance	Displacement
1	It is defined as the length of the actual path traversed by a body.	It is the shortest distance between two points in which the body moves.
2	It is a scalar quantity.	It is a vector quantity.
3	It is always positive.	It can be negative, positive or zero.
4	Distance can be equal to or greater than the displacement.	Displacement can be equal to or less than the distance.
5	Distance travelled is not a unique path between two points.	Displacement is a unique path between two points.
6	The distance between two points gives full information of the type of path followed by the body.	Displacement between two points does not give full information of the type of path followed by the body.
7	Distance never decreases with time. For a moving body, it is never zero.	Displacement can decrease with time. For a moving body, it can be zero.
8	Distance in SI unit is measured in meter.	Displacement in SI unit is measured in meter.

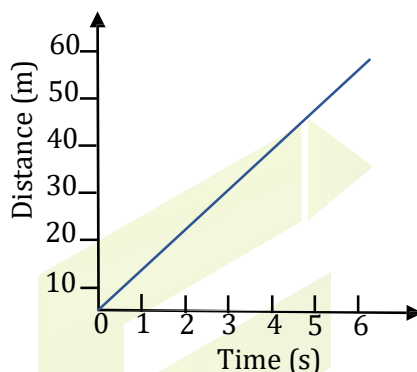
4. Uniform and non-uniform motion

A moving body may cover equal distances in equal intervals of time or different distances in equal intervals of time. On the basis of above assumption, the motion of a body can be classified as uniform motion and non-uniform motion.

Uniform motion

When a body covers equal distances in equal intervals of time, however small may be the time intervals, in a particular direction, the body is said to describe **a uniform motion**. (see figure).

Time (in second)	0	1	2	3	4	5	6
Distance covered (in metre)	0	10	20	30	40	50	60



Distance-time graph for uniform motion

☞ Uniform motion always takes place in a straight line.

In uniform motion, velocity of particle remains constant i.e., its magnitude as well as direction are constant. In uniform motion, average speed/velocity is equal to instantaneous speed/velocity at any point of time.

Examples of uniform motion

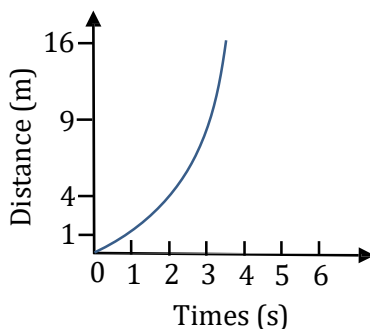
- (i) An aeroplane flying at a speed of 600 km/h along north.
- (ii) A train running at a speed of 120 km/h along east.
- (iii) Light energy travelling at a speed of 3×10^8 m/s in vacuum.

Non-uniform motion

When a body covers unequal distances in equal intervals of time, the body is said to be moving with **a non-uniform motion**. (see figure)

☞ Any motion along a curved path is always non-uniform motion. Also, any motion in which particle changes its direction is also non-uniform motion.

Time (in second)	0	1	2	3	4
Distance (in metre)	0	1	4	9	16



Distance-time graph for non-uniform motion

Examples of non-uniform motion

- (i) An aeroplane running on a runway before taking off.
- (ii) A freely falling stone under the action of gravity.
- (iii) When the brakes are applied to a moving car.
- (iv) A fan rotating with constant speed is also a non-uniform motion.

5. Speed

Speed of a body is the distance travelled by the body per unit time. The rate of change of distance is called speed.

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

If a body covers a distance s in time t then speed,

$$v = \frac{s}{t}$$

Unit

In SI system : m/s or ms⁻¹ ; In CGS system : cm/s or cms⁻¹

A commonly used unit of speed is km/h or kmh⁻¹.

Speed is a **scalar** quantity because it has magnitude but no direction. Speed is always taken positive.

Uniform speed

When a body covers equal distances in equal intervals of time, the body is to be moving with a uniform speed or constant speed.

Examples : (i) A train running with a speed of 120 km/h.
(ii) An aeroplane flying with a speed of 600 km/h.

Non-uniform speed

When a body covers unequal distances in equal intervals of time, the body is said to be moving with non-uniform speed or variable speed.

Examples : (i) A car running on a busy road. (ii) An aeroplane landing on a runway.

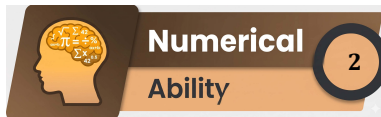
Average speed

The average speed of the body in a given time interval is defined as the total distance travelled, divided by total time taken.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

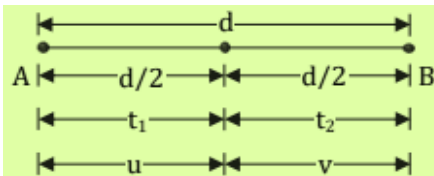
$$\frac{1\text{km}}{\text{h}} = \frac{1000\text{m}}{60 \times 60\text{s}} = \frac{5}{18} \text{ m/s}$$

$$\text{Speed in m/s} = \frac{5}{18} \times \text{speed in km/h ;} \quad \text{Km h}^{-1} \xleftarrow{\times \frac{5}{18}} \xrightarrow{\times \frac{18}{5}} \text{ms}^{-1}$$



1. A car travels first half distance with a uniform speed u and next half distance with a uniform speed v . Find its average speed.

Solution



Numerical Ability 2 (1)

$$\text{Total distance} = \frac{d}{2} + \frac{d}{2} = d \quad [\text{See figure}]$$

$$\text{Total time} = t_1 + t_2 = t$$

$$\therefore t_1 = \frac{d/2}{u} \quad \dots(i) \quad \left[t = \frac{s}{v} \right]$$

$$t_2 = \frac{d/2}{v} \quad \dots(ii)$$

$$V_{av} = \frac{d}{t} = \frac{d}{t_1 + t_2} \quad \text{Putting the value of equation (i) and (ii),}$$

$$V_{av} = \frac{d}{\frac{d/2}{u} + \frac{d/2}{v}} = \frac{d}{\frac{d}{2} \left(\frac{1}{u} + \frac{1}{v} \right)} \quad (\text{Taking } d/2 \text{ common out of bracket in the denominator})$$

$$= \frac{2}{\frac{1}{u} + \frac{1}{v}} = \frac{2uv}{u+v}$$

$$\therefore V_{av} = \frac{2uv}{u+v}$$

Decode the problem

Identify the type of motion

Uniform motion ✓ / Non-uniform motion

Identify the formula

Average speed =

$$\frac{\text{Total distance travelled}}{\text{Total time taken}}$$

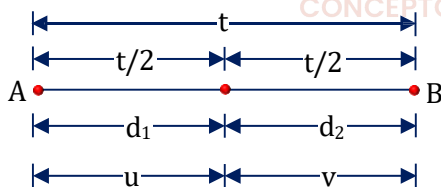
2. A car travels first half time with a uniform speed u and next half time with a uniform speed v . Find its average speed.

Solution

Since the car travels with a constant speed therefore, to find the distance the formula to be used is

Distance = speed $\times$ time.

$$d_1 = u \times \frac{t}{2}, \quad d_2 = v \times \frac{t}{2} \quad [\text{See figure}]$$



Numerical ability 2 (2)

$$\text{Total distance } d = d_1 + d_2 = u \times \frac{t}{2} + v \times \frac{t}{2}$$

$$d = \frac{ut}{2} + \frac{vt}{2} = \frac{t}{2}(u+v)$$

Decode the problem

Identify the type of motion

Uniform motion ✓ / Non-uniform motion

Identify the formula

Average speed =

$$\frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Total time = t

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

$$V_{av} = \frac{\frac{t}{2}(u+v)}{t}$$

$$\therefore V_{av} = \frac{u+v}{2}$$

Instantaneous speed

The speed of a body at any particular instant of time during its motion is called the instantaneous speed of the body. It is measured by **speedometer** in vehicles.



Quick Tips

- ★ If body covers equal distances $x_1 = x_2 = x$ (let), with different speeds i.e., u and v , then directly use the given formula,

$$V_{\text{average}} = \frac{2uv}{u+v}$$

- ★ If the two time intervals are same i.e., $t_1 = t_2 = t$ (let), speeds are different then directly use the given formula,

$$V_{\text{average}} = \frac{v+u}{2}$$

- ★ If the total distance or displacement is given then use the given formula,

$$V_{\text{avg speed}} = \frac{\text{Total distance}}{\text{Total time}} \quad V_{\text{avg velocity}} = \frac{\text{Total displacement}}{\text{Total time}}$$



Numerical Ability

3

1. The average speed of a bicycle, an athlete and a car are 18 km/h, 7 m/s and 2 km/min. respectively. Which of the three is the fastest and which is the slowest?

Solution

$$18 \text{ km/h} = \frac{18\text{km}}{1 \text{ h}} = \frac{18000\text{m}}{3600\text{s}} = 5 \text{ m/s}$$

$$2 \text{ km/min} = \frac{2\text{km}}{1 \text{ min.}} = \frac{2000\text{m}}{60\text{s}} = 33.3 \text{ m/s}$$

Thus, the average speeds of the bicycle, the athlete and the car are 5 m/s, 7 m/s and 33.3 m/s respectively. So the car is the fastest, and the bicycle is the slowest.

Decode the problem

To find the fastest and slowest speed, we need to convert all the given speeds in SI units.

Identify the formula

$$\text{Km h}^{-1} \xrightarrow[\times \frac{5}{18}]{\times \frac{18}{5}} \text{ms}^{-1}$$

2. On a 120 km track, a train travels the first 30 km with a uniform speed of 30 km/h. How fast must the train travel the next 90 km so as to average 60 km/h for the entire trip?

Solution

Given; Total distance $d = 120$ km, Average speed $V_{av} = 60$ km/h, Total time $= t = ?$

$$V_{av} = \frac{d}{t} \text{ or } t = \frac{d}{V_{av}}$$

Putting the values,

$$t = \frac{120\text{km}}{60\text{km/h}} = 2 \text{ h} \quad \dots(i)$$

Distance travelled in first part of trip, $d_1 = 30$ km, Speed in first part of the trip,

$$v_1 = 30 \text{ km/h}$$

Time taken in first part of trip, $t_1 = ?$, $t_1 = \frac{d_1}{v_1}$

$$\text{Putting the values, } t_1 = \frac{30\text{km}}{30\text{km/h}} = 1 \text{ h}$$

Time taken to complete second part of the trip, $t_2 = t - t_1 = 2 - 1 = 1 \text{ h}$

Distance to be covered in second part of the trip, $d_2 = 90$ km,

Required speed in second part, $v_2 = ?$

$$\text{Speed} = \frac{\text{distance}}{\text{time}},$$

$$\therefore v_2 = \frac{d_2}{t_2} = \frac{90\text{km}}{1\text{h}} = 90 \text{ km/h}$$

Check the unit

Use km/hr as the distance is given in km and time is given in hrs.

3. A bus going from Kota to Jaipur passed the 100 km, 160 km and 220 km points at 10:30 am, 11:30 am and 1:30 pm. Find the average speed of the bus during each of the following intervals: (a) 10.30 am to 11.30 am, (b) 11.30 am to 1.30 pm and (c) 10.30 am to 1.30 pm.

Solution

- (a) The distance covered between 10:30 am and 11:30 am is $160 \text{ km} - 100 \text{ km} = 60 \text{ km}$. The time interval is 1 hour. The average speed during this interval is –

$$v_1 = \frac{60\text{km}}{1\text{h}} = 60 \text{ km/h}$$

Decode the problem

Identify the type of motion

Uniform motion✓ /

Non-uniform motion

Identify the formula

Average speed =

Total distance travelled

Total time taken

Decode the problem

Identify the type of motion

Uniform motion✓ /

Non-uniform motion

Identify the formula

Average speed =

Total distance travelled

Total time taken

- (b) The distance covered between 11:30 am and 1:30 pm is $220 \text{ km} - 160 \text{ km} = 60 \text{ km}$.
The time interval is 2 hours. The average speed during this interval is –

$$v_2 = \frac{60 \text{ km}}{2 \text{ h}} = 30 \text{ km/h}$$

- (c) The distance covered between 10:30 am and 1:30 pm is $220 \text{ km} - 100 \text{ km} = 120 \text{ km}$.
The time interval is 3 hours. The average speed during this interval is –

$$v_3 = \frac{120 \text{ km}}{3 \text{ h}} = 40 \text{ km/h}$$

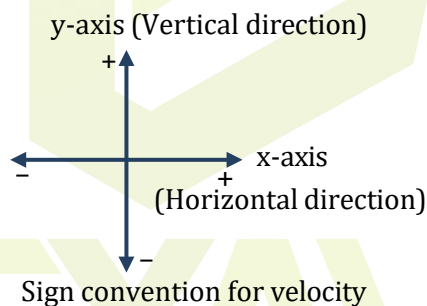
6. Velocity

The velocity of a body is the displacement of a body per unit time.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

The displacement covered by a body per unit time or the speed of a body in specified direction is called **velocity**.

☞ Velocity is a **vector** quantity. It can be positive, negative or zero (see figure).



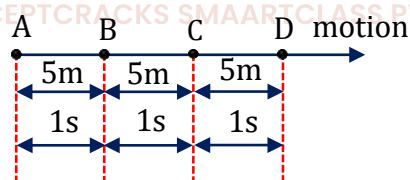
Unit

In SI system : m/s or ms^{-1}

In CGS system : cm/s or cms^{-1}

Uniform velocity

When a body covers equal displacements in equal intervals of time in a particular direction, the body is said to be moving with a uniform velocity (see figure).



Body moving with uniform velocity

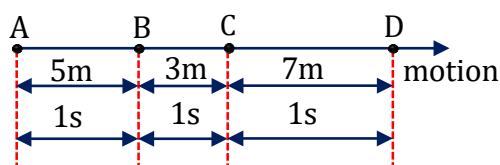
Conditions for uniform velocity

- The body must cover equal displacements in equal intervals of time.
- The direction of motion of the body should not change.

Example : A train running towards south with a speed of 120 km/h .

Non-uniform velocity/variable velocity

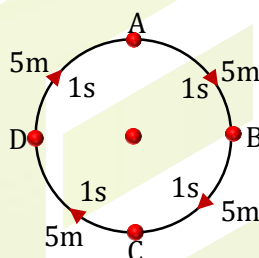
When a body covers unequal displacements in equal intervals of time, the body is said to be moving with variable velocity (see figure).



Body moving with non-uniform velocity

When a body covers equal distances in equal intervals of time, but its direction changes, then the body is said to be moving with variable velocity.

Example : In circular motion, a particle may have constant speed but its direction changes continuously thus, its velocity is non-uniform (see figure).



Body moving with variable velocity

Conditions for variable velocity

- (i) It should cover unequal displacements in equal intervals of time.
- (ii) It should cover equal distances in equal intervals of time but its direction must change.

Examples

- (i) A car running towards north on a busy road has a variable velocity as the displacement covered by it per unit time changes with change in the road condition.
- (ii) The blades of a rotating ceiling fan, a person running around a circular track with constant speed etc. are examples of variable velocity.

Average velocity

Total displacement of a particle divided by total time taken is called average velocity.

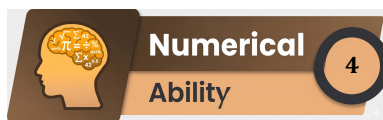
$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

$$V_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

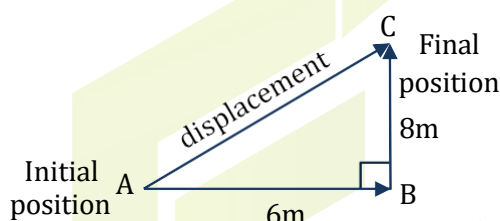
- ☞ Average speed is always greater than or equal to magnitude of average velocity. Average speed is equal to average velocity when particle moves in a straight line without change in direction.

Instantaneous velocity

The velocity of a body at any particular instant of time during its motion is called the instantaneous velocity of the body.



A particle moves along a path ABC as shown in figure. The time taken during the journey is 2 seconds. Find the average speed and average velocity during the journey.



Solution

Total distance travelled,

$$s = AB + BC = 6 + 8 = 14 \text{ m}$$

Average speed,

$$V_{av} = \frac{s}{t} = \frac{14}{2} = 7 \text{ m/s}$$

To find the displacement, we should know the initial and the final position of the particle.

In a right angle triangle,

$$\text{Hypotenuse (AC)} = \sqrt{(\text{Perpendicular})^2 + (\text{Base})^2}$$

$$\text{Here, displacement} = \vec{s} = AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10 \text{ m}$$

∴ Average velocity,

$$\vec{V}_{av} = \frac{\vec{s}}{t} = \frac{10}{2} = 5 \text{ m/s}$$

☞ Here, average speed is greater than average velocity because the direction of particle changes during motion.

Decode the problem

Identify the type of motion

Uniform motion ✓ /

Non-uniform motion

Identify the formula

Average speed =

$$\frac{\text{Total distance travelled}}{\text{Total time taken}}$$

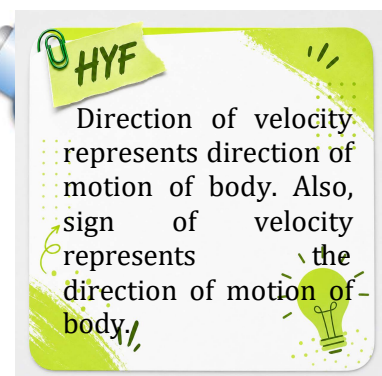
Identify the formula for Average velocity =

$$\frac{\text{Total displacement travelled}}{\text{Total time taken}}$$

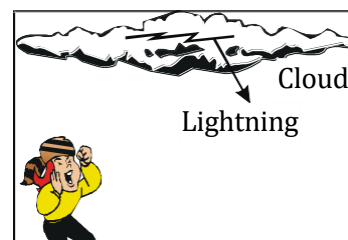
Identify the initial position and the final position and the shortest distance between them.



- Two buses depart from Jaipur, one going to Kota and one to Delhi. Each bus travels at a speed of 30 m/s. Do they have equal velocities? Explain.
- One of the following statements is incorrect. (a) The car travelled around the track at a constant velocity. (b) The car travelled around the track at a constant speed. Which statement is incorrect and why?



On any cloudy day or night, sometimes we see frequent lightning and hear the sound of thunder. The sound of thunder takes some time to reach us after we see the lightning. This is because light travels with very high speed ($= 3 \times 10^8$ m/s) while sound travels with much lower speed ($= 346$ m/s).



Active physics 1

To measure the distance (s) to the nearest point of thunder, we first measure the time interval (t) between the lightning and thunder as observed by us using a stop watch (see figure).

Thus, distance = speed of sound $\times$ time, or $s = v \times t = 346 \times t$

For example, if the time interval is 3 s then,

$$s = 346 \times 3 = 1038 \text{ m}$$

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Comparison between speed and velocity

	Speed	Velocity
1	It is defined as the rate of change of distance.	It is defined as the rate of change of displacement.
2	It is a scalar quantity.	It is a vector quantity.
3	It is always positive.	It can be negative, positive or zero.
4	Speed is velocity without direction.	Velocity is directed speed.
5	Speed in SI unit is measured in ms^{-1} .	Velocity in SI unit is measured in ms^{-1} .



1. No, they do not have equal velocities. This is because the velocities have same magnitude but they do not have same direction.
2. Statement (a) is incorrect. When a car travels around a track, its velocity is not constant because its direction changes continuously.

7. Acceleration

In uniform motion, the velocity remains constant with time. Thus, the change in velocity for any time interval is zero. But, in non-uniform motion, velocity changes with time. Thus, the change in velocity for any time interval has a non zero value.

In non-uniform motion, a new physical quantity called 'acceleration' is used.

The rate of change of velocity of a moving body with time is called **acceleration**.

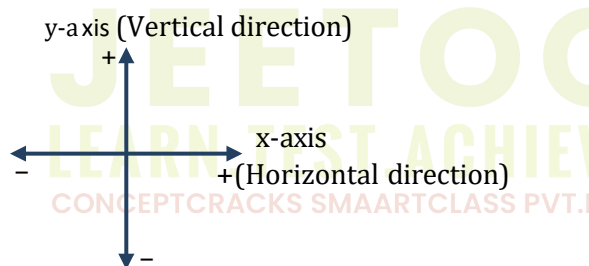
$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken for change}}$$

But, change in velocity = final velocity – initial velocity.

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time taken for change}} = a = \frac{v - u}{t}$$

If body moves with uniform velocity, then $v = u$ and then acceleration is zero i.e. $a = 0$.

☞ Acceleration is a vector quantity. It can be negative, positive or zero (see figure).



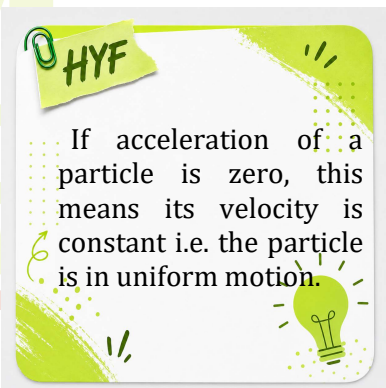
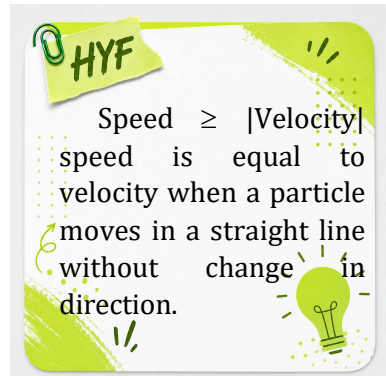
Sign convention for acceleration

Unit of acceleration

In SI system : m/s^2 or ms^{-2} , In CGS system : cm/s^2 or cms^{-2}

If the velocity of an object increases with time, such a motion is called '**accelerated motion**'. In such motion, acceleration 'a' is considered positive for numerical problems.

Example : An object starts from rest and its velocity goes on increasing with time.

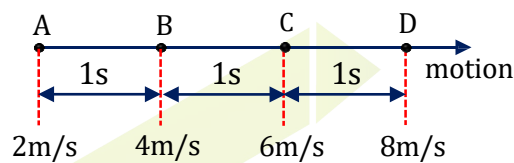


If velocity of an object decreases with time, such a motion is called '**retarded motion**'. In such a motion, acceleration is called '**retardation**' or '**deceleration**' and it is considered negative for numerical problems.

Example : A vehicle in motion is stopped by applying brakes.

Uniform acceleration

When a body moving in a straight line undergoes equal changes of velocity in equal intervals of time, the body is said to be moving with a uniform acceleration. Also, uniform acceleration means an acceleration having a constant magnitude and a constant direction (see figure).



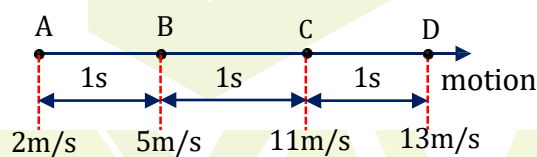
Uniformly accelerating body

Examples : (i) Motion of a freely falling body.

(ii) Motion of a ball rolling down on an inclined plane.

Non-uniform acceleration or variable acceleration

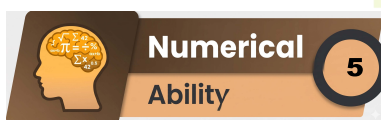
When a body undergoes unequal changes of velocity in equal intervals of time, the body is said to be moving with non-uniform acceleration (see figure).



Non-uniformly accelerating body

Examples : (i) The motion of a bus leaving or entering the bus stop.

(ii) A car moving on a busy road has non-uniform acceleration.



**Numerical
Ability**

5

1. A scooter acquires a speed of 36 km/h in 10 s after the start. Calculate the acceleration of the scooter.

Solution

Given, Initial velocity, $u = 0$. Final velocity, $v = 36 \text{ km/h}$
 $36 \times \frac{1}{60} \text{ km} \quad 36 \times \frac{5}{18} \text{ m}$

$$= \frac{1 \text{ km}}{1 \text{ h}} = \frac{18 \text{ m}}{18 \text{ s}} = 10 \text{ m/s}$$

Time taken, $t = 10 \text{ s}$

$$a = \frac{10 \text{ m/s} - 0 \text{ m/s}}{10 \text{ s}} = \frac{10 \text{ m/s}}{10 \text{ s}} = 1 \text{ m/s}^2$$

Acceleration = **1 m/s²**

Decode the problem

Identify the type of motion

Uniform motion /

Non-uniform motion ✓

Identify the formula

$$a = \frac{v - u}{t}$$

2. The velocity of a runner is 10 m/s. After 4 seconds, his velocity becomes 20 m/s. What is the value of acceleration?

Solution

Given, initial velocity, $u = 10 \text{ m/s}$, final velocity, $v = 20 \text{ m/s}$;
time, $t = 4 \text{ seconds}$

$$a = \frac{v - u}{t} = \frac{20 \text{ m/s} - 10 \text{ m/s}}{4 \text{ s}}$$

$$= \frac{10 \text{ m/s}}{4 \text{ s}}$$

$$= 2.5 \text{ m/s}^2$$

Acceleration = **2.5 m/s²**

Decode the problem

Identify the type of motion

Uniform motion /

Non-uniform motion✓

Identify the formula

$$a = \frac{v - u}{t}$$

8. **Equations of uniformly accelerated motion**

When an object moves with a uniform acceleration, its motion is called '**uniformly accelerated motion**'.

In case of uniformly accelerated motion, the average velocity is given by

$$V_{av} = \frac{v + u}{2}, \text{ here } v = \text{final velocity, } u = \text{initial velocity}$$

These equations give the relationship between initial velocity, final velocity, time taken, acceleration and distance travelled by the body.

First equation of motion

A body having an initial velocity 'u' acted upon by a uniform acceleration 'a' for time 't' such that final velocity of the body is 'v'.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time taken}}$$

$$a = \frac{v - u}{t} \text{ or } v - u = at, \text{ or } \boxed{v = u + at}$$

Second equation of motion

It gives the distance travelled by a body in time 't'.

A body having an initial velocity 'u' acted upon by a uniform acceleration 'a' for time 't' such that final velocity of the body is 'v' and the distance covered is 's'.

$$V_{av} = \frac{v + u}{2}$$

Distance covered = average velocity $\times$ time taken

$$s = \left(\frac{v + u}{2} \right) \times t \quad \dots (1)$$

but $v = u + at$ (from first equation of motion) ... (2)

Using (1) & (2), we get, $s = \left(\frac{u + at + u}{2} \right) t = \left(\frac{2u}{2} + \frac{at}{2} \right) t = \left(u + \frac{at}{2} \right) t$ or $\boxed{s = ut + \frac{1}{2}at^2}$

Third equation of motion

A body having an initial velocity 'u' moving with a uniform acceleration 'a' for time 't' such that final velocity is 'v' and the distance covered is 's'.

$$V_{av} = \frac{v + u}{2}$$

Distance covered = average velocity $\times$ time taken

$$s = \left(\frac{v + u}{2} \right) \times t \quad \dots (1)$$

Now, $v = u + at$, or $v - u = at$

or $t = \left(\frac{v - u}{a} \right) \quad \dots (2)$

From (1) & (2), we get,

$$s = \left(\frac{v + u}{2} \right) \left(\frac{v - u}{a} \right) = \frac{v^2 - u^2}{2a} \quad \text{or} \quad 2as = v^2 - u^2$$



- ★ While deriving the second equation of motion we eliminate final velocity (v) and replace it with (u + at) as the second equation of motion gives the relation between the distance/displacement, initial velocity, acceleration and time.
- ★ While deriving third equation of motion, we eliminate time (t) and replace it with $\left(\frac{v - u}{a} \right)$ as the third equation of motion gives the relation between the distance/displacement, initial velocity, acceleration and final velocity.



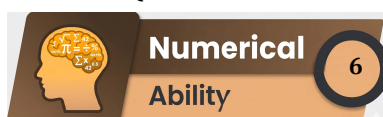
In your everyday life, you come across a range of motions in which

- (a) Acceleration is in the direction of motion.
- (b) Acceleration is against the direction of motion.
- (c) Acceleration is uniform.
- (d) Acceleration is non-uniform.

Identify one example each of the above types of motion.

Solution

- (a) While increasing the speed of vehicle using the accelerator, the acceleration is in the direction of motion.
- (b) While applying brakes of a vehicle, its speed decreases with time. Here, the acceleration is against the direction of motion.
- (c) Motion of a particle under gravity has a uniform acceleration ($g = 9.8 \text{ m/s}^2$, vertically downwards).
- (d) Motion of a car in a crowded traffic has non-uniform acceleration as its speed varies (increases or decreases) as per the need.



1. A car accelerates uniformly from 18 kmh^{-1} to 36 kmh^{-1} in 5 s.

Calculate (i) acceleration, (ii) distance covered by the car in that time.

Solution

Given, $u = 18 \text{ km/hr} = 5 \text{ m/s}$;

$v = 36 \text{ km/h} = 10 \text{ m/sec}$; $t = 5 \text{ s}$

- (i) We use first equation of motion

$$a = \frac{v - u}{t} = \frac{10 \text{ m/s} - 5 \text{ m/s}}{5 \text{ s}}$$

$$a = 1 \text{ m/s}^2$$

- (ii) We use second equation of motion

$$s = ut + \frac{1}{2}at^2$$

$$s = 5 \times 5 + \frac{1}{2} \times 1 \times 5 \times 5$$

$$s = 25 \text{ m} + 12.5 \text{ m} = 37.50 \text{ m}$$

Decode the problem

Identify the type of motion

Uniform motion / Non-uniform motion ✓

Identify the equation

$$v = u + at \quad \checkmark$$

(for part (i), since, u , v and t are given)

$$s = ut + \frac{1}{2}at^2 \quad \checkmark$$

(for part (ii), since, u , a and t are given)

$$v^2 = u^2 + 2as$$

2. A car initially at rest starts moving with a constant acceleration of 0.5 m s^{-2} and travels a distance of 25 m. Find its final velocity.

Solution

Given, $u = 0$, $a = 0.5 \text{ m/s}^2$, $s = 25 \text{ m}$

We use third equation of motion

$$v^2 = u^2 + 2as$$

$$v^2 = (0)^2 + 2 \times 0.5 \times 25 = 25$$

$$v = \sqrt{25} = 5 \text{ m/s}$$

Decode the problem

Identify the equation

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as \quad \checkmark$$

(since, u , a and s are given)

3. A racing car has a uniform acceleration of 5 m/s^2 . What distance will it cover in 20 s after starting from rest?

Solution

The car starts from the position of rest. So, its initial velocity (u) is zero. Then, $u = 0 \text{ m/s}$, $a = 5 \text{ m/s}^2$, $t = 20 \text{ s}$, $s = ?$

For uniformly accelerated motion, we have three equations from which we use one equation.

$$s = (0 \text{ m/s} \times 20 \text{ s}) + \frac{1}{2} \times (5 \text{ m/s}^2) \times (20 \text{ s})^2$$

$$\text{or } s = 0 \text{ m} + \frac{1}{2} \times 5 \times 20 \times 20$$

$$= 5 \times 10 \times 20 = \mathbf{1000 \text{ m} = 1 \text{ km}}$$

So, the car will cover a distance of 1000 m (or 1 km) in 20 s.

4. A body starts from rest and has a velocity of 3 m/s . If it has a uniform acceleration of 10 m/s^2 . Find the distance travelled by in 3 seconds.

Solution

The body starts from the position of rest. So, its initial velocity (u) is zero. Then, $u = 0 \text{ m/s}$, $a = 10 \text{ m/s}^2$, $t = 3 \text{ s}$, $s = ?$

$$s = (0 \text{ m/s} \times 3 \text{ s}) + \frac{1}{2} \times (10 \text{ m/s}^2) \times (3 \text{ s})^2$$

$$\text{or } s = 0 \text{ m} + \frac{1}{2} \times 10 \text{ m} \times 3 \times 3 = 5 \text{ m} \times 3 \times 3 = \mathbf{45 \text{ m}}$$

5. If a body has a velocity of 50 m/s , and it takes 4 seconds to travel a distance of 240 m, then what is the acceleration of the body?

Solution

Given, initial velocity, $u = 50 \text{ m/s}$,
time, $t = 4 \text{ s}$, distance, $s = 240 \text{ m}$,
acceleration, $a = ?$

$$240 = (50 \times 4) + \left(\frac{1}{2} \times a \times 16\right);$$

$$240 = 200 + 8a \Rightarrow 40 = 8a$$

$$a = \mathbf{5 \text{ m/s}^2}$$

Decode the problem

Identify the type of motion

Uniform motion / Non-uniform motion ✓

Identify the equation

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2 \checkmark$$

Decode the problem

Identify the equation

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2 \checkmark$$

$$v^2 = u^2 + 2as$$

(since, u , a and s are given)

Decode the problem

Identify the type of motion

Uniform motion / Non-uniform motion ✓

Identify the equation

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2 \checkmark$$

$$v^2 = u^2 + 2as$$

(since, u , a and s are given)

6. The speed of a car is reduced from 54 km/hr to 36 km/hr in a certain time during which it travelled 125 m. Calculate the acceleration of the car.

Solution

Given, Initial velocity,

$$u = 54 \text{ km/h} = \frac{54 \times 1 \text{ km}}{1 \text{ h}} = \frac{54 \times 5 \text{ m}}{18 \text{ s}} \\ = 15 \text{ m/s}$$

Final velocity,

$$v = 36 \text{ km/h} \\ = \frac{36 \times 1 \text{ km}}{1 \text{ h}} = \frac{36 \times 5 \text{ m}}{18 \text{ s}} = 10 \text{ m/s}$$

Distance, $s = 125 \text{ m}$; acceleration = ?

$$a = \frac{v^2 - u^2}{2s} = \frac{(10)^2 - (15)^2}{2 \times 125} \\ = \frac{100 - 225}{250} = \frac{-125}{250} = -0.5$$

acceleration = - 0.5 m/s²

Decode the problem

Convert speed from km/hr to m/sec.

Identify the type of motion

Uniform motion / Non-uniform motion ✓

Identify the equation

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$v^2 = u^2 + 2as \checkmark$$

(since, u , a and s are given)

7. A train starting from rest attains a speed of 90 km/hr. If the acceleration of the train is 0.5 m/s², how much distance will it travel?

Solution

Decode the problem Given, initial velocity, $u = 0$; acceleration, $a = 0.5 \text{ m/s}^2$

Identify the type of motion Final velocity, $v = 90 \text{ km/h} = \frac{90 \times 1 \text{ km}}{1 \text{ h}} = \frac{90 \times 5 \text{ m}}{18 \text{ s}} = 25 \text{ m/s}$

Uniform motion / Non-uniform motion ✓

Identify the formula

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$v^2 = u^2 + 2as \checkmark$$

For uniformly accelerated motion, we have three equations from which we use one equation.

(Since u , v and a are given)

We use third equation of motion

$$v^2 = u^2 + 2as$$

$$(25 \text{ m/s})^2 = 0 + 2 \times 0.5 \times s \Rightarrow 625 = 0 + s$$

$$s = 625 \text{ m}$$

8. The driver of a train travelling at 40 ms^{-1} applies the brakes as a train enters a station. The train slows down at a rate of 2 ms^{-2} . How long will the train travel before it stops?

Solution

Given, $u = 40 \text{ m/s}$, $v = 0$, $a = -2 \text{ m/s}^2$ (negative sign is taken as the velocity of object decreases with time and hence it is a retarded motion)

$$(0)^2 = (40 \text{ m/s})^2 + 2 \times (-2 \text{ m/s}^2) \times s$$

$$4 \text{ m/s}^2 \times s = 40 \times 40 \text{ m}^2/\text{s}^2$$

$$s = \frac{1600}{4}$$

$$s = 400 \text{ m}$$

Decode the problem

Identify the type of motion

Uniform motion / Non-uniform motion ✓

Identify the formula

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as \quad \checkmark$$



Quick Tips

- ★ The second equation of motion always tells the relation between the distance/displacement and time. If in a given problem these quantities are mentioned, then apply second equation of motion.
- ★ The third equation of motion always tells the relation between the distance/displacement and final velocity. If in a given problem these quantities are mentioned, then apply third equation of motion.
- ★ Squares of few numbers are given below. Learning these will make the calculations quick.

Number	Square	Number	Square	Number	Square	Number	Square
1	1	6	36	11	121	16	256
2	4	7	49	12	144	17	289
3	9	8	64	13	169	18	234
4	16	9	81	14	196	19	361
5	25	10	100	15	225	20	400



Be Alert !

- ★ Displacement, velocity and acceleration are vector quantities. Always take care of the sign convention of these quantities.
- ★ If the final velocity is greater than the initial velocity, then acceleration is taken positive and if the final velocity is less than the initial velocity, then acceleration is taken negative in the numerical.

- ★ Always take care of the units while solving the numerical. All the units should be in the same system of units.

Example : If the initial velocity of a body is 20 m/s, acceleration is 1 m/s², time is 2 min, then find the final velocity.

Here, the time is to be converted into seconds, so that all the quantities are in same system of units and the final answer is also in the correct units.

9. Graphical representation of motion

Graph

A graph is a line, straight or curved, showing the relation between two variable quantities, of which one varies as a result of the change in the other.

The quantity which changes independently is called **independent variable** and the one which changes as a result of the change in the other is called **dependent variable**.

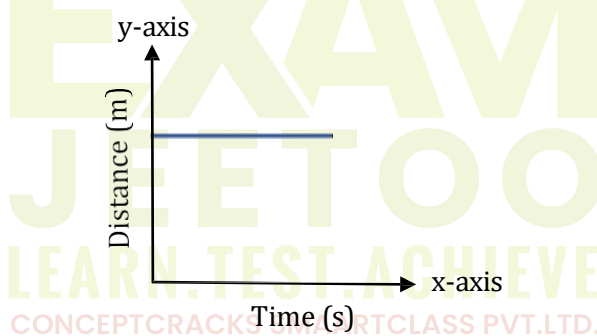
Distance/displacement–time graph

This graph is plotted between the time taken and the distance covered. The time is taken along the x-axis and the distance covered is taken along the y-axis.

- The slope of the distance-time graph gives the speed of the body.
- The slope of the displacement-time graph gives the velocity of the body.

When the body is at rest

When position of the body does not change with time then it is said to be stationary. The distance-time graph of such a body is a straight line parallel to x-axis (see figure).



s-t graph for a body at rest

When the body is in uniform motion

In uniform motion,

$$v = \text{constant.}$$

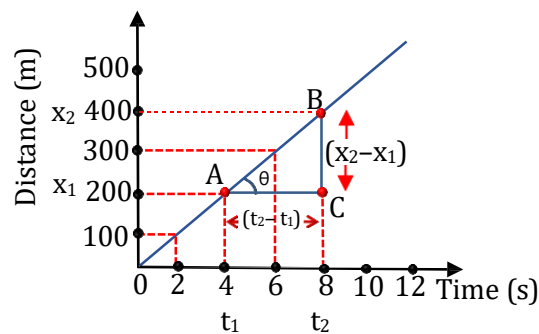
Now, $s = v \times t$ Since v is constant

$\therefore s \propto t$, thus, the distance-time graph of such a body is a straight line, inclined to x-axis.

$$\text{Slope} = \frac{\text{measure on y-axis}}{\text{measure on x-axis}} = \tan \theta$$

$$\text{or Slope} = \frac{x_2 - x_1}{t_2 - t_1} = v$$

Thus, slope of distance-time graph gives speed of the body (see figure).

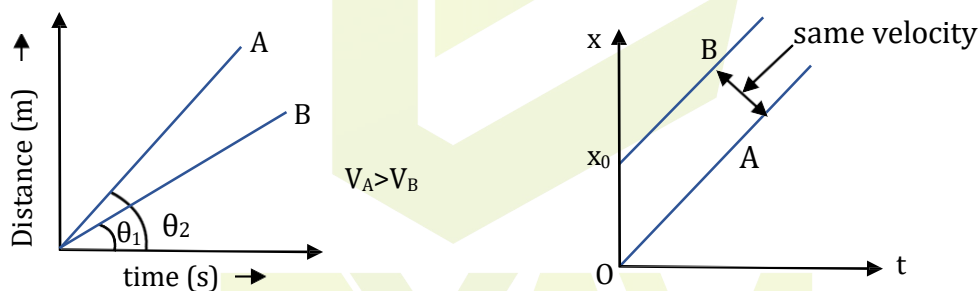


s-t graph for a body in uniform motion ($a = 0$)

- ☞ In a distance-time graph, more the slope of the graph of an object in motion, more will be its speed and vice-versa.

Example : In given graph (see figure a) speed of particle A is greater than the speed of particle B because slope of graph of particle A is greater than slope of graph of particle B.

- ☞ In the given graph (see figure b), speed of particle A is equal to speed of particle B because graphs of both have same slope.

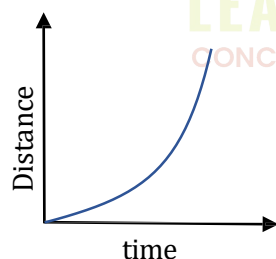


(a) Two particles moving with different velocities

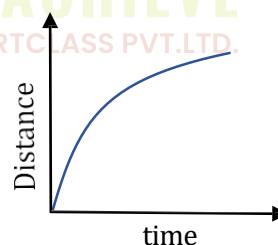
(b) Two particles moving with same velocities

When the body is in non-uniform motion

In this case, distance-time graph is a curve. For example, in uniformly accelerated motion, $s = ut + \frac{1}{2}at^2$. Since $s \propto t^2$, definitely the graph is not a straight line, it is a curve (see figure).



s-t graph of an accelerated motion
(speed increasing with time)



s-t graph of a retarded motion
(speed decreasing with time)

- ☞ A distance-time graph can never be parallel to y-axis (representing distance) because this line has inclination of 90° , and slope = $\tan \theta = \tan 90^\circ = \text{infinite}$, which means infinite speed; it is impossible.

Speed/velocity-time graph

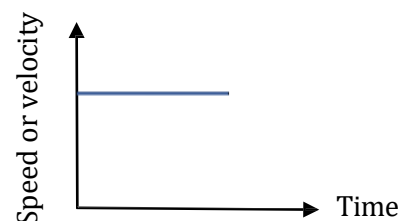
The variation in velocity with time for an object moving in a straight line can be represented by a velocity-time graph. In this graph, time is represented along the x-axis and velocity is represented along the y-axis.

The slope of the speed/velocity-time graph gives the acceleration of the body.

Area enclosed under a speed-time graph or velocity-time graph gives the distance covered by the body.

When the body is in uniform motion

Here, the body moves with constant velocity. The speed or velocity of the body is uniform, hence the magnitude remains same. The graph is a straight line parallel to x-axis (time-axis). Since the velocity is uniform, its acceleration is zero. The slope of the graph in this case is zero (see figure).

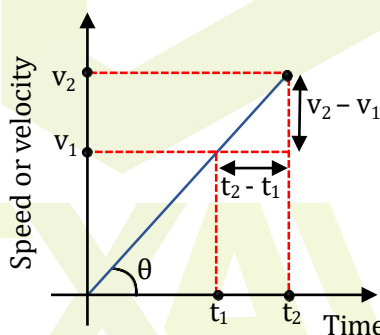


v-t graph for an object in uniform motion

When the body is moving with a uniform acceleration

$$\text{Slope} = \frac{v_2 - v_1}{t_2 - t_1} = \tan \theta = a$$

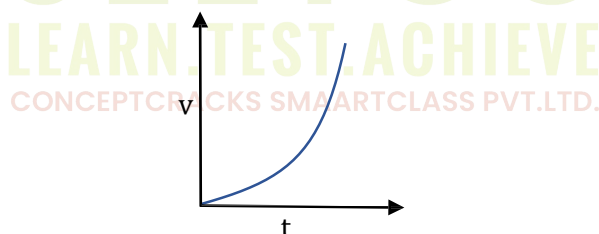
Thus, slope of v-t graph gives acceleration of a body (see figure).



v-t graph for an object moving with uniform acceleration

When the body is moving with a non-uniform (variable) acceleration.

In this case, the speed or velocity-time graph is not a straight line, but is a curve (see figure.)



v-t graph for an object moving with non-uniform acceleration

- ☞ Speed or velocity-time graph line can never be parallel to y-axis (speed axis), because inclination becomes 90° , then $\tan 90^\circ$ is infinite; ie. infinite acceleration is impossible.

Distance from speed or velocity-time graph

As distance = speed $\times$ time, hence the distance can be calculated from speed or velocity-time graph.

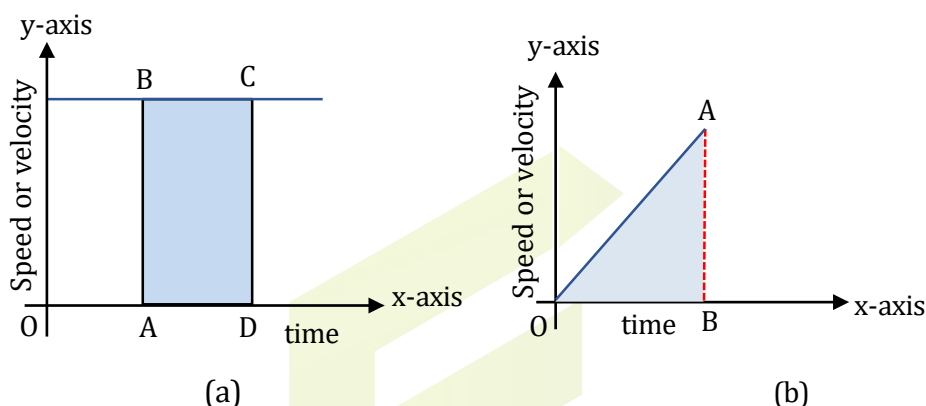
When speed or velocity is uniform (constant)

Distance = Area of rectangle ABCD = AB × AD [see figure (a)]

When acceleration is uniform (constant)

Distance or displacement = Area of right-angled triangle OAB [see figure (b)]

$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \text{OB} \times \text{BA}$$



Area under v-t graph gives distance travelled by a body



- ★ The unit of distance (m) is used to find the area under the curve of speed/velocity time graph.

Distance = Area under the curve in the above graph = length (quantity on y-axis) × breadth (quantity on x-axis).

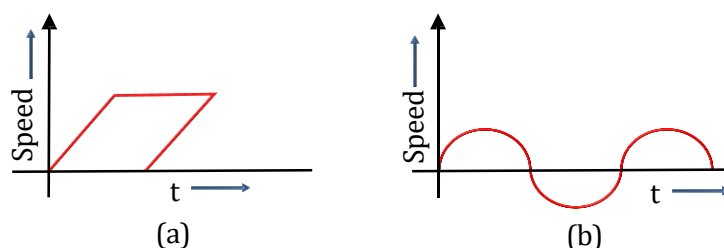
$$\text{Distance} = \text{Velocity} \times \text{time} = \frac{\text{m}}{\text{s}} \times \text{s} = \text{m}$$



Check your
Concepts

2

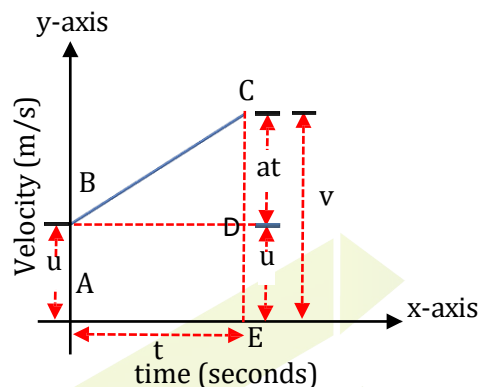
- What is represented by the slope of v-t graph?
- Give velocity-time graph for a motion in which velocity and acceleration are in the same direction.
- State with reasons which of these cannot possibly represent one-dimensional motion of a particle (see figure).



Check your concepts 2 (3)

10. Equations of motion by graphical method

Figure below represents a velocity-time graph, in which AB represents the initial velocity u , CE represents final velocity v , such that the change in velocity is represented by CD, which takes place in time t , represented by AE (see figure).



An object moving with certain initial velocity u and attains final velocity v after time t .

Derivation of $v = u + at$

Acceleration = slope of the graph line BC

$$a = \frac{CD}{BD} = \frac{CE - DE}{BD} \quad \text{or} \quad a = \frac{v - u}{t} \quad \left[\begin{array}{l} \because DE = AB = u \\ \because BD = AE = t \\ \because CE = v \end{array} \right]$$

$$v - u = at$$

$$\boxed{v = u + at}$$

Derivation of $s = ut + \frac{1}{2}at^2$

Distance travelled = Area of rectangle ABDE + Area of triangle BCD

$$\begin{aligned} &= AB \times AE + \frac{1}{2} (BD \times CD) = u \times t + \frac{1}{2} [t \times (v - u)] \quad \left[\begin{array}{l} \because BD = AE = t \\ \because CD = CE - DE = v - u \end{array} \right] \\ &= u \times t + \frac{1}{2} [t \times (u + at - u)] \quad [\because v = u + at] \end{aligned}$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

Derivation of $v^2 = u^2 + 2as$

From the velocity-time graph, distance covered = Area of trapezium ABCE

$$\Rightarrow s = \frac{1}{2} (AB + CE) \times AE \quad \therefore s = \frac{1}{2} (u + v) \times t \quad \dots(i)$$

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}}$$

$$a = \frac{v - u}{t} \quad \therefore t = \frac{v - u}{a} \quad \dots(ii)$$

Substituting the value of t in equation (i)

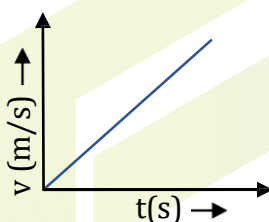
$$s = \frac{(v+u)}{2} \times \frac{(v-u)}{a} \quad [\because A^2 - B^2 = (A+B) \times (A-B)]$$

$$s = \frac{v^2 - u^2}{2a},$$

$$\text{or } v^2 - u^2 = 2as, \text{ or } \boxed{v^2 = u^2 + 2as}$$



1. Slope of v-t graph gives 'acceleration'.
2. Since velocity and acceleration are in same direction, the velocity of particle increases with time (see figure).



Check your answers 2

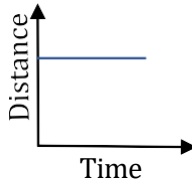
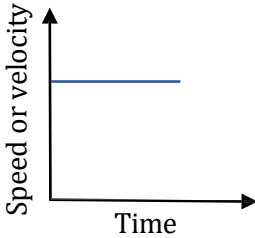
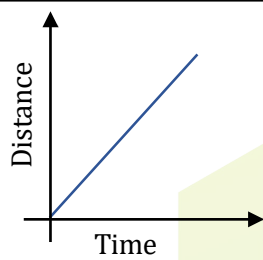
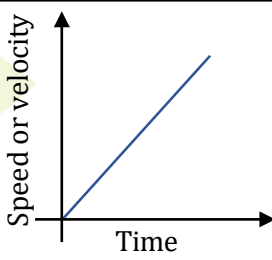
3. (1) Graph (a) is incorrect as time always increases; it cannot be reversed.
- (2) Graph (b) is also incorrect as speed is always positive, it can never be negative.



- ★ Area of rectangle = length $\times$ breadth
- ★ Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
- ★ Area of trapezium = $\frac{1}{2} \times \text{sum of length of parallel sides} \times \text{height or distance between parallel sides. } \left[\frac{1}{2}(a+b) \times h \right]$



Always take care while reading the graph. The first step is always to observe the quantities mentioned on x-axis and y-axis. The shape of the graph may appear the same for any two graphs, but they can provide information representing different types of motion. A quick comparison for your better understanding is given below.

Distance-time graph	Velocity-time graph
 s-t graph for a body at rest	 v-t graph for an object in uniform motion
 s-t graph for a body in uniform motion ($a = 0$)	 v-t graph for an object moving with uniform acceleration



Quick Tips

- ★ The slope of distance/displacement time graph gives the speed/velocity of the body.
- ★ The slope of speed/velocity time graph gives the acceleration of the body.
- ★ More the slope of distance/displacement time graph more will be the speed/velocity of the moving body.



Numerical Ability

7

- The distance covered by a moving body was recorded after every 1 s for 4 s. The data is given in the table below. Represent it graphically and find the speed of the body from the graph.

Distance (m)	0	3	6	9	12
Time (s)	0	1	2	3	4

Solution

The body covers equal distance in equal interval of time. The distance-time graph of a body in uniform motion is a straight line. The horizontal axis represents time, while the vertical axis represents distance (see figure).

$$\text{Speed} = \frac{OG}{OF} = \frac{6\text{m}}{2\text{s}} = 3 \text{ m/s}$$

Decode the problem

Identify the graph

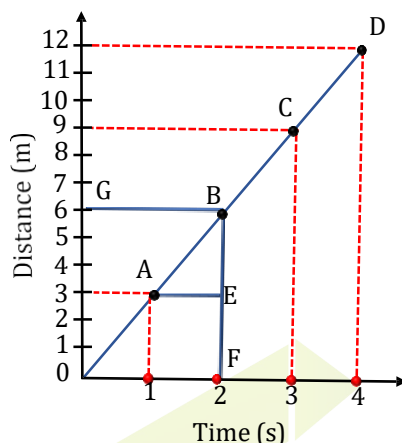
According to data, body covers equal distance in equal interval of time. So graph will be distance time graph of uniform motion.

Apply the formula

Speed = Slope =

$\frac{\text{measure on y-axis}}{\text{measure on x-axis}}$

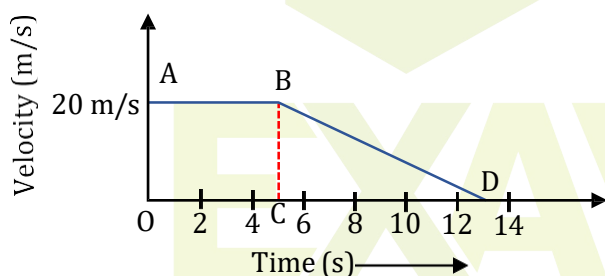
If we study the graph carefully, we will see that BE/AE would also give the speed of the body. BE gives the distance travelled in the time interval AE , which is 1 s.



Numerical ability 7 (1)

2. A car travels with a uniform velocity of 20 ms^{-1} for 5s. The brakes are then applied and the car is uniformly retarded. It comes to rest in further 8 s. Draw a graph of velocity against time. Use this graph to find :
- the distance travelled in first 5 s,
 - the distance travelled after the brakes are applied,

Solution



Numerical ability 7 (2)

- (i) The distance travelled in first 5 seconds = area of rectangle OABC
 area of rectangle = length $\times$ breadth

$$= 20 \text{ m/s} \times 5 = \mathbf{100 \text{ m}}$$

- (ii) The distance travelled in last 8 seconds (from 5 sec to 13 sec)

$$= \text{area of the triangle BCD} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times (13 - 5) \text{ s} \times 20 \text{ m/s}$$

$$= \frac{1}{2} \times 8 \text{ s} \times 20 \text{ m/s} = \mathbf{80 \text{ m}}$$

Decode the problem

Draw the graph

Take velocity on y-axis and time of x-axis.

When the car travels with a uniform velocity, the graph is a straight line graph parallel to x-axis. When the car is uniformly retarded, the velocity decreases and becomes zero. The graph for this is a straight line with a decreasing slope.

Identify the formula

The area under the curve in a velocity time graph gives the distance travelled by the car.

3. In the previous question, find the acceleration during the first 5 s and last 8 s.

Solution

Acceleration during first 5 second is zero as velocity is constant.

$$a = \frac{v - u}{t} = \frac{20 - 20}{5} \\ = 0 \text{ m/s}^2$$

For the last 8 seconds, the initial velocity of the car is 20 m/s and the final velocity is 0.

In the last 8 seconds, acceleration,

$$a = \frac{v - u}{t} = \frac{0 - 20}{13 - 5} = \frac{20}{8} \\ = -2.5 \text{ m/s}^2$$

Decode the problem

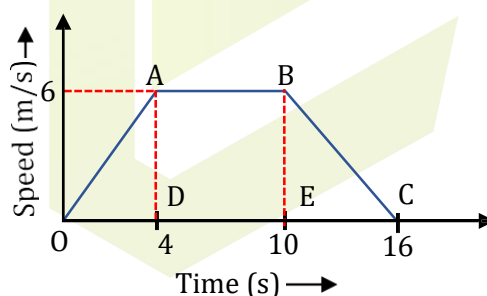
Study the graph

- (i) For first 5 seconds initial and final velocity is same.
- (ii) For the last 8 seconds, $u = 20 \text{ m/s}$ and $v = 0$.

Identify the formula

$$a = \frac{v - u}{t}$$

4. Study the speed-time graph of a body given here and answer the following questions.



Numerical ability 7 (4)

- (a) What type of motion is represented by OA?
- (b) What type of motion is represented by AB?
- (c) What type of motion is represented by BC?

Solution

The y-axis of the graph denotes speed and the x-axis denotes time.

- (a) OA is a straight line graph between speed and time. The slope of speed-time graph represent acceleration. Thus OA represent uniform acceleration.
- (b) AB is a straight line parallel to the time axis. The speed is constant from A to B. Thus AB represent uniform speed.
- (c) BC is a straight-line graph between speed and time (downward slope). BC represents uniform retardation or negative acceleration.

Decode the problem

Study the graph

- (a) Graph represents object moving with uniform acceleration
- (b) Graph represents object is in uniform motion
- (c) Graph represent object moving with uniform retardation

5. In the previous question, find the acceleration, retardation and the distance travelled by the body.

Solutions

$$(i) \quad a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{6 - 0}{4 - 0}$$

$$= 1.5 \text{ m/s}^2$$

$$(ii) \quad a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{0 - 6}{16 - 10}$$

$$= -1 \text{ m/s}^2$$

(iii) Distance travelled by the body = area of shaded portion = area of triangle OAD + area of rectangle DABE + area of triangle BEC

$$= \left(\frac{1}{2} \times 4 \times 6 \right) + (6 \times 6) + \left(\frac{1}{2} \times 6 \times 6 \right)$$

$$= 66 \text{ m}$$

Decode the problem

Study the graph

- (i) In first 4 seconds speed increases 0 to 6 m/s.
- (ii) In the last 6 seconds speed decreases from 6 m/s to 0.
- (iii) The area under the curve of a speed time graph gives the distance travelled by the body.

Identify the formula

- (i) Acceleration = slope of line OA

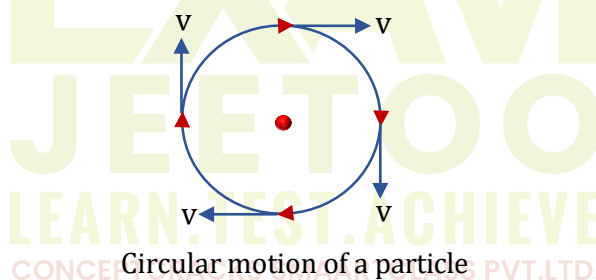
$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

- (ii) Acceleration = slope of line BC

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

11. Circular motion

When a particle moves along a circular path, its motion is called '**circular motion**' (see figure).



☞ In a circular motion, velocity of a particle is tangential to the circular path.

If the body covers equal distances along the circumference of the circle, in equal intervals of time, then motion is said to be a **uniform circular motion**. When a body moves along a circular path, then its direction of motion changes continuously. Thus, **a circular motion is always a non-uniform motion**.

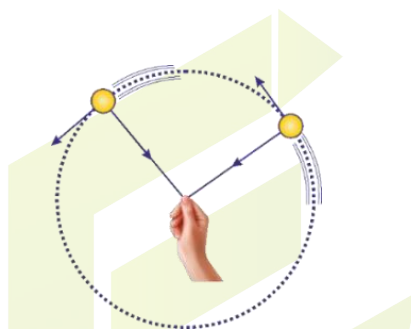
☞ A uniform circular motion is a motion in which speed remains constant but direction of velocity changes continuously.

Examples of uniform circular motion

- (i) An athlete running on a circular track with constant speed.
- (ii) Motion of tips of the second hand, minute hand and hour hand of a wrist watch.



Take a piece of thread and tie a small piece of stone at one of its ends. Move the stone to describe a circular path with constant speed by holding the thread at the other end (see figure).



Active Physics 3

When the stone is released, it will move in a direction tangential to the circular path. If we release the stone from different positions, direction of movement of stone will be different for different positions. But, it is always tangential to the circular path at the position where the stone is released.

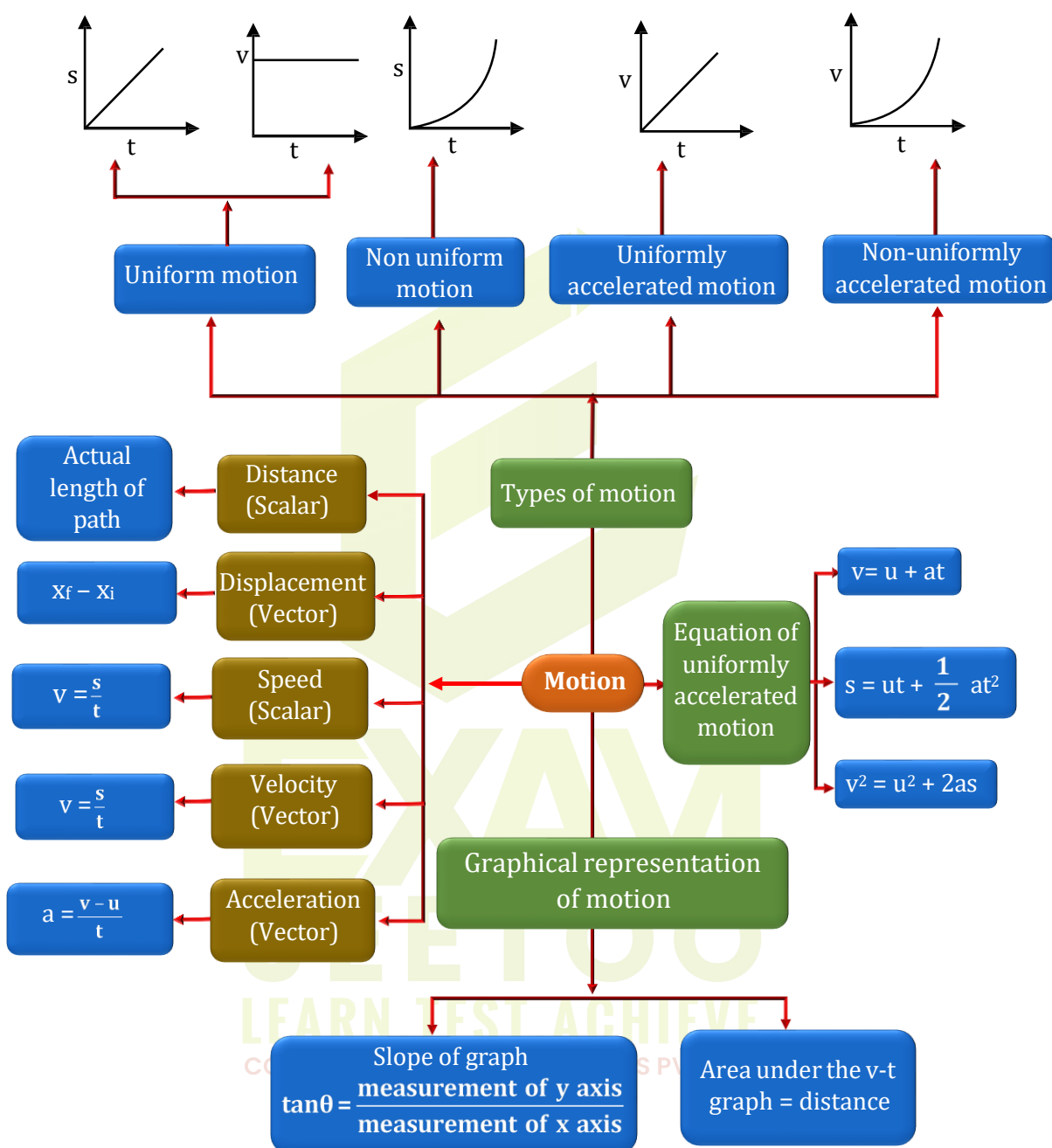
Circular motion is always accelerated motion. The direction of velocity of the particle changes continuously due to the change in the direction of motion.

The circumference of a circle of radius r is given by $2\pi r$.

If a particle takes t seconds to go one around the circular path of radius r , the speed is given by

$$v = \frac{2\pi r}{t}$$

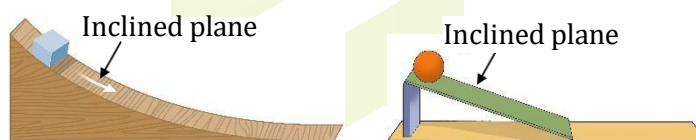
Concept Map



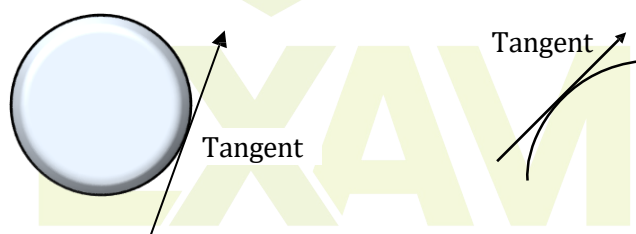
Important terms, formula, unit, quantity			
Quantity	Formula	Unit	Scalar/Vector
Distance		m	Scalar
Displacement		m	Vector
Speed/Velocity	$v = \frac{s}{t}$	m/s	Scalar/Vector
Average Speed	$\frac{\text{Total distance}}{\text{Total time}}$	m/s	Scalar
Average Speed (If the TWO distances are same)	$V_{\text{avg}} = \frac{2v_1 v_2}{v_1 + v_2}$	m/s	Scalar
Average Speed (If the TWO time intervals are same)	$V_{\text{avg}} = \frac{v_1 + v_2}{2}$	m/s	Scalar
Average Velocity	$\frac{\text{Total displacement}}{\text{Total time}}$	m/s	Vector
Acceleration	$a = \frac{v - u}{t}$	m/s ²	Vector
Slope of a straight line graph	$\text{Slope} = \tan\theta = \frac{\text{perpendicular}}{\text{base}}$	-	-
Equations of motion			
First equation of motion	$v = u + at$		
Second equation of motion	$s = ut + \frac{1}{2} at^2$		
Third equation of motion	$v^2 = u^2 + 2as$		

Some Basic Terms

1. **Linear** – In straight line.
2. **Dimension** – Something as length, breadth, depth, height.
3. **Magnitude** – A numerical quantity or value.
For example: in 10 meters, 10 is the magnitude; in 15 m/s, 15 is the magnitude etc.
4. **Initial** – Beginning or starting.
5. **Circumference** – The distance around a circle (formula = $2\pi r$, where r is the radius of the circle).
6. **Vacuum** – A space which has no matter.
7. **Constant** – Something which does not change.
8. **Variable** – A quantity which is able to assume different numerical values.
9. **Inclined** – A sloping ramp; for example – slide.



10. **Infinite** – Limitless or endless in space.
11. **Tangent** – A straight line that touches a curved surface at a point, but if extended does not cross it at that point.



12. **Trapezium** – A quadrilateral which has two sides parallel to each other.



SOLVED EXAMPLES

1. In a long-distance race, the athletes were expected to take four rounds of the track such that the line of finish was same as the line of start. Suppose the length of the track was 200 m.

- (i) What is the total distance to be covered by the athletes?
- (ii) What is the displacement of the athletes when they touch the finish line?
- (iii) Is the motion of the athletes uniform or non-uniform?
- (iv) Is the displacement of an athlete and the distance moved by him at the end of the race equal?

Solution

- (i) Total distance covered = $4 \times 200 \text{ m} = 800 \text{ m}$
- (ii) As the athletes finish at the starting line,
Displacement = final position - initial position = $x_A - x_A = 0$
- (iii) Motion is non-uniform as the direction of motion of the athlete is changing while running on the track.
- (iv) Displacement and distance moved are not equal.

2. Nisha swims in a 90 m long pool. She covers 180 m in one minute by swimming from one end to the other and back along the same straight path. Find the average speed and average velocity of Nisha.

Solution

Total distance = 180 m

Total displacement = 0

Time taken, $t = 1 \text{ min.} = 60 \text{ s}$

Average speed (V_{av}) = $\frac{\text{total distance}}{\text{total time taken}}$

$$V_{av} = \frac{180 \text{ m}}{60 \text{ s}} = 3 \text{ m/s}$$

Average velocity

$$(V_{av}) = \frac{\text{total displacement}}{\text{total time taken}}$$

$$V_{av} = \frac{0}{60 \text{ s}} = 0 \text{ m/s}$$

3. The average speed of a bicycle, an athlete and a car are 18 km/h, 7 m/s and 2 km/min. respectively. Which of the three is the fastest and which is the slowest?

Solution

$$18 \text{ km/h} = \frac{18 \text{ km}}{1 \text{ h}} = \frac{18000 \text{ m}}{3600 \text{ s}} = 5 \text{ m/s}$$

$$2 \text{ km/min} = \frac{2 \text{ km}}{1 \text{ min.}} = \frac{2000 \text{ m}}{60 \text{ s}} = 33.3 \text{ m/s}$$

Thus, the average speeds of the bicycle, the athlete and the car are 5 m/s, 7 m/s and 33.3 m/s respectively. So the car is the fastest, and the bicycle is the slowest.

4. A boy starting from rest, starts running and attains a velocity of 6 ms^{-1} in 30 s. Then he slows down uniformly to 4 ms^{-1} in next 5 s. Calculate his acceleration in both cases.

Solution

In the first case

initial velocity, $u = 0$;

final velocity, $v = 6 \text{ ms}^{-1}$;

time, $t = 30 \text{ s}$.

$$a = \frac{v - u}{t} = \frac{(6 - 0) \text{ ms}^{-1}}{30 \text{ s}} = 0.2 \text{ ms}^{-2}$$

In the second case :

initial velocity, $u = 6 \text{ ms}^{-1}$;

final velocity, $v = 4 \text{ ms}^{-1}$;

time, $t = 5 \text{ s}$.

$$a = \frac{v - u}{t} = \frac{(4 - 6) \text{ ms}^{-1}}{5 \text{ s}} = -0.4 \text{ ms}^{-2}$$

NCERT Basics : Class 9

5. An object undergoes an acceleration of 8 m/s^2 starting from rest. Find the distance travelled in 1 second.

Solution

Given, acceleration, $a = 8 \text{ m/s}^2$, time, $t = 1 \text{ s}$
initial velocity, $u = 0$, distance travelled, $s = ?$

$$\text{As } s = ut + \frac{1}{2}at^2, s = 0 \times 1 + \frac{1}{2} \times 8(1)^2 = 4 \text{ m}$$

6. A car acquires a velocity of 72 km h^{-1} in 10 s starting from rest. Calculate :
(i) the acceleration,
(ii) the distance travelled in this time.

Solution

Given, initial velocity, $u = 0$

Final velocity, $v = 72 \text{ km h}^{-1}$

$$= \frac{72 \times 1000 \text{ m}}{60 \times 60 \text{ s}} = 20 \text{ ms}^{-1}$$

Time taken $t = 10 \text{ s}$

(i) Acceleration

$$a = \frac{v - u}{t} = \frac{(20 - 0) \text{ ms}^{-1}}{10 \text{ s}} = 2 \text{ ms}^{-2}$$

(ii) Distance travelled, $S = ut + \frac{1}{2}at^2$

$$= 0 + \frac{1}{2} \times 2 \times (10)^2 = 100 \text{ m}$$

7. An athlete runs of 20 min at a form speed of 36 km/hr . At what speed should he run for the next 10 min. so that the average speed will be 54 km/hr .

Solution

$$\text{Total time} = 20 \text{ min} + 10 \text{ min} = 30 \text{ min} = \frac{1}{2} \text{ hr}$$

$$\text{Average speed} = 54 \text{ km/hr}$$

$$\text{Distance covered in 30 min} = 54 \times \frac{1}{2} = 27 \text{ km}$$

$$\text{Distance covered in first 20 min} = 36 \times \frac{1}{3} = 12 \text{ km}$$

$$\text{Athlete needs to cover} = (27 - 12) \text{ km} = 15 \text{ km in next 10 min.}$$

$$\text{so, required speed} = \frac{15}{\frac{10}{60}} = \frac{15 \times 60}{10} = 90 \text{ km/hr}$$

8. A body starts from rest and attains the speed of 20 m/s . If the acceleration of the body is 4 m/s^2 then what is the distance travelled by the body.

Solution

Given,

initial velocity, $u = 0$;

final velocity, $v = 20 \text{ ms}^{-1}$;

acceleration, $a = 4 \text{ m/s}^2$

$$v^2 = u^2 + 2as, (20)^2 = (0)^2 + 2 \times 4 \times s$$

$$s = \frac{20 \times 20}{2 \times 4} = 50 \text{ m}$$

9. A motorcycle moving with a speed of 5 m/s is subjected to an acceleration of 0.2 m/s^2 . Calculate the speed of the motorcycle after 10 second, and the distance travelled in this time.

Solution

Given, $u = 5 \text{ m/s}$, $a = 0.2 \text{ m/s}^2$

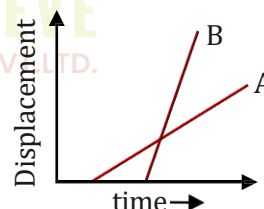
$v = ?$, $t = 10$, $s = ?$

$$\text{From } v = u + at, v = 5 + 0.2 \times 10 = 7 \text{ m/s}$$

from $v^2 - u^2 = 2as$,

$$s = \frac{v^2 - u^2}{2a} = \frac{7^2 - 5^2}{2 \times 0.2} = 60 \text{ m/s}$$

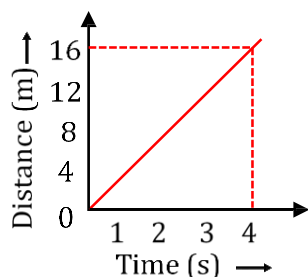
10. Figure shows distance-time graph of two objects A and B. Which object is moving with greater speed when both are moving?

**Solution**

The line for object B makes a larger angle with the time-axis. Its slope is larger than the slope of the line for object A. Thus, the speed of B is greater than that of A.

Physics

11. In the given distance-time graph find the speed of the body.

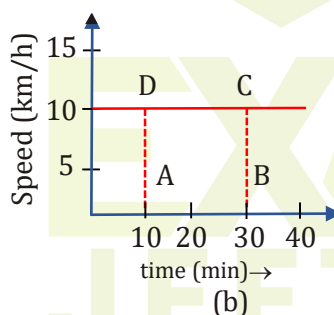
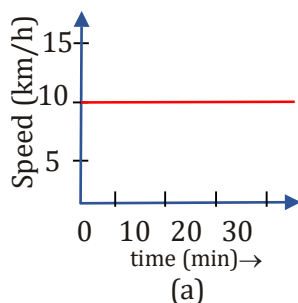
**Solution**

The slope of the distance-time graph gives the speed of body

$$\text{slope} = \text{speed} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{16 - 0}{4 - 0} = \frac{16}{4}$$

$$\text{speed} = 4 \text{ m/s}$$

12. Figure represents the speed-time graph for a particle. Find the distance covered by the particle between $t = 10$ min. and $t = 30$ min.

**Solution**

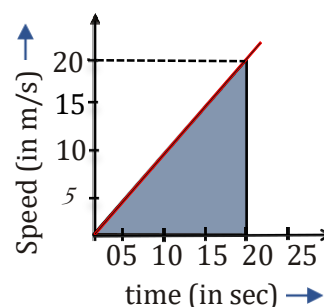
We draw perpendicular lines from the 10-minute point and the 30-minute point to the line of graph (see fig. (b)). The distance covered is equal to the area of the rectangle ABCD. Its value is

$$ABCD = (30 \text{ min.} - 10 \text{ min.}) \times (10 \text{ km/h})$$

$$= 20 \text{ min.} \times 10 \text{ km/h}$$

$$= \frac{20}{60} \text{ h} \times 10 \text{ km/h} = \frac{10}{3} \text{ km}$$

13. Find the distance covered by a particle during the time interval $t = 0$ to $t = 20$ s for which the speed-time graph is shown in figure.

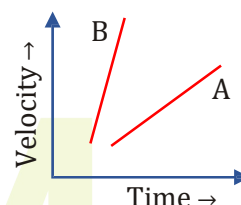
**Solution**

The distance covered in the time interval 0 to 20 s. is equal to the area of the shaded triangle. It is

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (20 \text{ s}) \times (20 \text{ m/s})$$

$$= 200 \text{ m.}$$

14. Figure shows the velocity-time graphs for two objects, A and B, moving along the same direction. Which object has greater acceleration?

**Solution**

The slope of the velocity-time graph of B is greater than that for A. Thus, the acceleration of B is greater than that of A.

15. A cyclist goes around a circular track of diameter 42 m in 5 minutes. Calculate his speed.

Solution

Given, Diameter = 42 m;

$$\text{Radius} = \frac{42}{2} \text{ m} = 21 \text{ m}$$

Time, $t = 5 \text{ min} = 5 \times 60 \text{ sec} = 300 \text{ sec}$

Velocity, v ?

$$v = \frac{2\pi r}{t} = 2 \times \frac{22}{7} \times \frac{21}{300} = 0.44 \text{ m/s}$$